Problem Set 1

Feel free to collaborate with classmates on problem sets.

This problem set deals with a Robinson Crusoe economy with two factors of production and two commodities.

Let there be two factors: land denoted T, and labor denoted L. The resource endowment of T is T^o; the resource endowment of L is L^o .

Let there be two goods, x and y.

Robinson has a utility function u(x, y). There is no utility from leisure.

The prevailing wage rate of labor is w, and the rental rate on land is r.

Good x is produced in a single firm by the production function $f(L^x,T^x) = x$, where L^x is L used to produce x, T^x is T used to produce x. $f(L^x,T^x) \geq 0$ for $L^x \geq 0$, $T^x \geq 0$; f(0,0)=0.

Good y is produced in a single firm by the production function $g(L^y,T^y) = y$ where L^y is L used to produce y, T^y is T used to produce y. $g(L^y,T^y) \geq 0$ for $L^y \geq 0$, $T^y \geq 0$; g(0,0)=0.

The price of good x is p^x . The price of good y is p^y . Profits of firm x are $\Pi^x=p^x f(L^x,T^x)-wL^x-rT^x$. Profits of firm y are $\Pi^y=p^y g(L^y,T^y)-wL^y-rT^y$.

Robinson's income then is $wL+rT+\Pi^x+\Pi^y$

Assume f, g, u, to be strictly concave, differentiable. Assume all solutions are interior solutions. Subscripts denote partial derivatives. That is, $u_x = (\partial u/\partial x)$ = marginal utility of x, ..., $f_L = (\partial f/\partial L)$ = marginal product of labor in x, ...

The production frontier consists of those x - y combinations that efficiently and fully utilize L^o and T^o in producing x and y. The marginal rate of transformation of x for y, $MRT_{x,y}$ is defined as -(dy/dx) along this frontier. $MRT_{x,y}$ is the additional y available from efficiently reallocating inputs of T and L to producing y while sacrificing one unit of x. At a technically efficient (efficient in allocation of inputs on the production side) allocation, we have

$$-(dy/dx) = MRT_{x,y} = (\partial y/\partial L^y)/(\partial x/\partial L^x) = g_L/f_L \ .$$

The marginal rate of transformation of x for y equals the ratio of marginal products.

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A (Pareto) efficient allocation in the economy is characterized by maximizing u(x,y) subject to the technology and resource constraints. Thus a Pareto efficient allocation corresponds to values of x,y,L^x,L^y,T^x,T^y maximizing the Lagrangian, Λ , with Lagrange multipliers a, b, c, d:

$$\Lambda = u(x,y) + a(x-f(L^x,T^x)) + b(y-g(L^y,T^y)) + c(L^o-L^x-L^y) + d(T^o-T^x-T^y) \qquad (1)$$

1. Differentiate Λ with respect to x, y, T^x, T^y, and set the derivatives equal to 0. That gives first order conditions for an extremum of Λ, a Pareto efficient allocation. Let (2) be your first order condition with respect to x, (3) with respect to y, (4) with respect to T^x, (5) with respect to T^y.

2. Show that Pareto efficiency requires that the marginal rate of substitution of x for y be the marginal rate of transformation (as computed with respect to T). That is, Pareto efficiency requires that

$$u_x/u_y = g_T/f_T \qquad (6)$$

Hint: You can demonstrate (6) by combining (2), (3), (4) and (5) appropriately. Explain in words what (6) means. Why does it make sense as an efficiency condition?

3. Differentiate Λ with respect to L^x, L^y, to characterize first order conditions for a Pareto efficient allocation of labor.

4. Repeat exercise **1** with respect to L. That is, show that Pareto efficiency requires that $u_x/u_y = g_L/f_L$.

5. Show that Pareto efficiency requires that marginal rates of technical substitution of L for T are the same for both firms. That is, Pareto efficiency requires $g_L/g_T = f_L/f_T$. Explain in words what this expression means.

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6. First order conditions for profit maximization and for utility maximization subject to budget constraint are:

$w = p^x f_L = p^y g_L$; (7)

$r = p^x f_T = p^y g_T$; (8)

$p^x/p^y = u_x/u_y$ (9).

These conditions (7), (8), (9), will be fulfilled in a competitive equilibrium. Show that these equilibrium conditions lead to fulfillment of the efficiency conditions in **2**, **3**, **4,** and **5**.